# What's Wrong with the Standard Model?

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03. Quantum mechanics, field theories, and special relativity

11. General theory of fields and particles

## Abstract

The Standard Model of Elementary Particles is based on elementary excitations classified by a single spin variable (non-negative half-integer) attributed to the irreducible representations of the rotation group in three dimensions. This feature originates from a theorem of Wigner [1,2], the derivation of which utilizes a homology between the symmetry group SL(2,C) and the Lorentz group, the homogeneous symmetry of Minkowski space. In recent work [3] it has been shown that the representations of SL(2,C) and the Lorentz group differ fundamentally, and that Wigner's theorem does not apply to the latter. Consequently, elementary fields have to be classified by two indices ($j$, $k$), each a non-negative half-integer. Correspondingly, there is also no need to introduce so-called internal symmetries, because the corresponding degrees of freedom follow directly from the second index. Furthermore, owing to the the product-nature of the representations, a mechanism can be suggested that interprets interaction as simple propagation between field factors of identical sub-index $j$ or $k$ [4]. This would remove the embarrassingly high number of free interaction parameters in the Standard Model.

## Symmetry of Space-Time

Space-time with coordinates $x_\mu$ ($\mu = 0,1,2,3$) are described in Minkowski's metric (sig. 1,-1,-1,-1). Physical processes are not influenced by changing the coordinate system such as shifting the origin (four translation symmetries), or Lorentz transformations (homogeneous changes which preserve the metric tensor). Fields are finite-dimensional vector quantities, defined for each point in space-time. A field at a given point experiences a transformation whenever the coordinate system in space-time is changed which signifies that it is a representation of the group of these transformations. As such it can be decomposed into a sum of irreducible representation. The irreducible fields correspond to elementary excitations (called particles). Thus, it is important to know the finite-dimensional representation of the Lorentz group. For the simpler situation of a Cartesian metric the homogeneous symmetry group is SO(4) with well known representations.

## The Cartesian case, SO(4)

Lie groups, here SO(4), can be generated by their infinitesimal generators which form a Lie algebra, here so(4), where they read, in terms of the coordinates $(t, x, y, z) = (x_0, x_1, x_2, x_3)$:

$A_x = z\,\partial^y - y\,\partial^z$ , and cyclic; infinitesimal rotations (1)

$$B_w = w\,\partial^t - t\,\partial^w \quad , w = x, y, z, \tag{2}$$

where the generators $B$ correspond to rotations in the $(t,w)$-plane.

The Lie algebra of so(4) obeys the commutation relations

$$[A_x, A_y] = A_z \quad ; \quad [B_x, B_y] = A_z \quad ; \quad [A_x, B_y] = [B_x, A_y] = B_z \quad \text{, and cyclic} \tag{3}$$

This algebra is a direct sum of two isomorphic Lie algebras with basis elements

$$J_w = \frac{A_w + B_w}{2} \quad \text{and} \quad K_w = \frac{A_w - B_w}{2} \quad \text{with commutation relations} \tag{4}$$

$$[L_x, L_y] = L_z \quad \text{, and cyclic, } L = J \text{ or } K. \tag{5}$$

Obviously, these two algebras are both isomorphic to the algebra of infinitesimal rotations in three dimensions, so(3). Consequently, the irreducible representations of SO(4) are direct products of two representations of the group SO(3), each of which is labeled by a single non-negative half integer. Consequently, the irreducible representations of SO(4) are labeled by a pair of indices $(j,k)$.

The vectors of representations of SO(4) carry no indication of any applied transformations, which is generally  the case for compact Lie groups. This seemingly unnecessary statement will become meaningful in comparison with the situation of the Lorentz group, summarized below.

## Minkowski Metric, the Lorentz Group SO(3,1)

For the Lorentz group the the situation is fundamentally different. The infinitesimal boost operators now read

$$B_w = w\,\partial^t + t\,\partial^w \quad , w = x, y, z. \tag{6}$$

In the corresponding Lie algebra so(3,1) one of the commutation relations (3) is changed to

$$[B_x, B_y] = -A_z \quad \text{, and cyclic.} \tag{7}$$

This has the consequence that this Lie algebra is not a direct sum of two equivalent sub-algebras any more. Nevertheless, so(4) and so(3,1) are real forms of their common complexification, and it is tempting to try to treat so(3,1) by introducing a imaginary time coordinate which recovers the commutation relations (3-5), although with imaginary boost operators $B$. This procedure lies at the basis of the Standard Model of Elementary Particles. In order to understand, why this is substantially different from considering the real form so(3,1) we summarize shortly the results of a analysis of the representations of the Lorentz group [3].

Representations of SO(3,1) are not equivalent to a unitary one and correspondingly the vector fields have a very different form. First of all, they have a time (multi-)component and three space (multi-)components, and furthermore, they carry a boost variable, $b$, of 3-vector type, which we must interpret as a quantity characterizing the field flow at the given space-time point. At this point we must remember that Lorentz transformations can always be separated into a product of a pure rotation followed by a pure boost.  Clearly, we can keep the product representations of SO(4), characterized by the pair of labels $(j, k)$, before the final boost is applied. This boost then modifies the original (time-)components of the representation vector and, in addition, generates additional (space-)components. Thus, the states of representations are to be be characterized by their final boost, the variable $b$ mentioned above. Zero b corresponds to the coordinate system in which the (local) field flow disappears.

# Standard Field Theory

In standard field theory, which is the basis of the Standard Model, it is assumed that the introduction of a imaginary time is a procedure which leads to the correct theory. This corresponds to replacing the actual symmetry of space-time, the Lorentz group, by the group SL(2,C). In this case, the local inner product of a field is given by multiplication with the complex conjugated vector. Postulating invariant transition probabilities Wigner derived a theorem which relates the behavior under a SL(2,C)-transformation $W$ of vectors of a representations $D(W)$ to the corresponding transformation in a situation where a shift of the origin of the coordinate system has been applied, as summarized below:

States of a representation which change phase by $e^{i(k^{\mu}u_{\mu})}$ under a shift of the coordinate origin by u, transform with the same matrix $D(W)$ as long as the 4-vector $Wk$ does not change its size. These matrices vary only within transformations of the so-called little group, a subgroup of Lorentz transformations which leave the vector $k$ unchanged. Furthermore, these matrices are identical for all vectors $k$ of the same size, such that one can choose a arbitrary representative among these $k$-vectors to obtain these matrices. Restricting ourselves to the case of massive particles, having time-like $k$, we may choose a representative without space components, as is commonly done. The only relevant transformations which leave this vector unchanged are pure rotations. Thus, it is argued that massive elementary particles can be classified by representations of the little group SO(3) alone, i.e. by a single index characterizing the spin.

However, there is already a alarming sign here, because the vectors of a representation have to be renormalized when changing to boosted states [2].

We argue that this reduction to SO(3) is an artifact of not treating representations of the Lorentz group properly as another real form of the complexified group of SO(4). This defect was pointed at in early days [5] but its repression seems to have been commonly accepted by now. Our criticism originates in the fact, that boosted vectors of a representation have a completely different transformation behavior from non-boosted ones [3], as emphasized above. Most conspicuously, they have additional (space-like) components compared to the non-boosted state. This is not born out within standard theory whenever one chooses a reference state with boosted $k$. We emphasize again that the origin of this discrepancy lies in considering representations of SL(2C) instead of the ones of the Lorentz group itself, a change suggested by the use of a imaginary time (see above), as well as by the 2 to 1 homology between SL(2C) and the Lorentz group [6].

# Conclusions

The principal conclusion of this note is that elementary fields must be classified by a pair of indices ($j$, $k$) instead of just a single one as commanded by Wigner's theorem and the ensuing reduction to representations of the little groups.

## *Internal Symmetries?*

A irritating feature of the Standard Model is that one had to introduce ad-hoc additional symmetry groups SU(3), SU(2), U(1), so-called internal symmetries, to fully classify elementary particles. This is unnecessary if one accepts that the correct representation theory provides two labels ($j$, $k$) for the classification [4]. As an example, a quark transforms according to the (½, 1)-representation, in which the three dimensions of the $k$=1 factor account for it's three colors.

Furthermore, the existence of a infinite sequence of irreducible representations (and corresponding elementary fields) contrasts fundamentally with all unification postulates.

### *Interactions*

The awkwardness of introducing symmetry-violating interaction terms has been pointed at before [7], and it must be questioned altogether whether the concept of separate interaction terms is meaningful at all. They introduce additional free parameters, the number of which increases without limits, given the fact that we must expect a unlimited number of elementary excitation modes corresponding to the possible irreducible representations of the Lorentz group.

A more meaningful concept is the inclusion of interaction within the mechanism of field propagation [4]. Spin-propagation operators are $j$- ($k$-) specific and it seems highly promising to postulate that they act on representation-vector factors of the same type ($j$ or $k$) in between fields of different representations. The case of $j = ½$, as a specific example, includes the electron (½, 0), the electric field (½, ½) and the quark (½, 1). Here the electric field is the only field (neglecting gravitation) that can exist as one-field solution (photon), and thus, act as a carrier at a distance. However, electron and quark could also interact directly (in the above mentioned sense of propagation at direct contact), and we may have to consider this as the mechanism, which is known as “weak interaction” [7]. Obviously, this interaction mechanism is specific to ($j$, $k$) product representations and cannot exist for fields classified by a single SO(3)-index.

Extending these ideas to gravitation, considered as the scalar field [4], one also arrives at a natural interpretation of the Higgs boson, avoiding the awkward introduction of a “ether”-type Higgs field [8].

## References


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